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# RESISTANCE TO OVERLAND FLOW DUE TO BED-LOAD TRANSPORT ON PLANE MOBILE BEDS

SHIXIONG HU[1]* AND ATHOL D. ABRAHAMS[2]

[1] *Department of Geography, East Stroudsburg University of Pennsylvania, East Stroudsburg, PA 18301, USA*
[2] *Department of Geography, University at Buffalo, The State University of New York, Buffalo, NY 14261, USA*

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## ABSTRACT

During bed-load transport by overland flow, momentum is transferred from the flow to the bed via grain collisions, resulting in a decrease in flow velocity and an increase in flow resistance, herein termed bed-load transport resistance. In overland flow on mobile plane beds, total flow resistance $f$ consists of grain resistance $f_g$ and bed-load transport resistance $f_{bt}$. In order to identify and evaluate the relative importance of the factors controlling $f_{bt}$, 38 flume experiments were performed on slopes of 2·7 and 5·5° using sediment with median diameters of 0·74 and 1·16 mm. All flows were supercritical and turbulent.

This study is an extension of a recent study by Gao and Abrahams (*Earth Surface Processes and Landforms* 2004, vol. 29, pp. 423–435). These authors found that $f_{bt}$ is controlled by three factors: sediment concentration $C$, dimensionless sediment diameter $D_*$, and relative submergence $h/D$, where $h$ is flow depth, $D$ is median sediment diameter. However, a new dimensional analysis identifies two additional factors: Froude number $F$ and slope $S$. Multiple regression analyses reveal (1) that these five factors together explain 97 per cent of the variance of $f_{bt}$, and (2) that $S$ controls $f_{bt}$ entirely through $C$. The variable $C$ is therefore redundant, and a new functional equation relating $f_{bt}$ to $D_*$, $h/D$, $S$ and $F$ is developed. This equation may be used to predict $f_{bt}$. An advantage of this equation is that it may be used to predict $f_{bt}$ without measuring bed-load transport rate. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS: flow resistance; bed-load transport; overland flow; hillslope processes

## INTRODUCTION

In bed-load transport, streamwise momentum is transferred from the fluid to the bed in a two-step process. First, momentum is transferred to the grains as they are lifted from the bed and accelerated downstream by the flow. Second, momentum is lost to the bed when the grains collide with the bed and grain streamwise velocity is reduced. This loss of momentum causes a decrease in streamwise flow velocity and an increase in flow resistance. Flow resistance may be measured by the Darcy–Weisbach friction factor

$$f = 8gSh/u^2 \tag{1}$$

where $g$ is the acceleration of gravity (m s$^{-2}$), $S$ is the energy slope, $h$ is the mean flow depth (m), and $u$ is the mean flow velocity (m s$^{-1}$). The increase in flow resistance caused by the movement of bed-load is herein termed bed-load transport resistance $f_{bt}$. As hillslope runoff is very sensitive to $f$, it is important to understand the contribution of $f_{bt}$ to $f$. Consequently, the overall goal of this study is to investigate the magnitude and controls of $f_{bt}$ in interrill overland flow.

Abrahams and Li (1998) investigated $f_{bt}$ in transitional and turbulent overland flows on a fixed plane bed coated with a single size of sand. Five discharges were studied. Hot-film anemometry was used to measure the velocity profiles of three flows: a clear-water flow, a flow with a relatively low volumetric sediment concentration (i.e. 0·0017), and a flow with a relatively high volumetric sediment concentration (i.e. 0·0127). It was estimated that from 83 to 90 per cent of the sediment was travelling as bed-load (Hu and Hui, 1996). In the sediment-laden flows, the near-bed velocities were smaller and the velocity profiles steeper than those in the

* Correspondence to: S. Hu, Department of Geography, East Stroudsburg University of Pennsylvania, East Stroudsburg, PA 18301, USA. E-mail: shu@po-box.esu.edu

Table I. Ranges of experimental data in relevant studies

| Property | Abrahams and Li (1998) | Song *et al.* (1998) | Gao and Abrahams* (2004) | This study |
|---|---|---|---|---|
| Bed condition | Fixed | Mobile | Mobile | Mobile |
| Sample size | 15 | 54 | 12 | 38 |
| $\beta$ (degree) | 2·7 | 0·29–0·86† | 2 | 2·5, 5·5 |
| $D$ (mm) | 0·74 | 12·3 | 7 | 0·74,1·16 |
| $h$ ($\times 10^{-2}$ m) | 0·45–0·70 | 8·4–25·3 | 2·6–7·1 | 0·32–0·81 |
| $h/D$ | 6·1–9·5† | 6·8–20·6† | 3·7–10·1 | 3·6–10·5 |
| $u$ (m s$^{-1}$) | 0·373–0·555 | 0·9–1·18 | 0·65–1·24 | 0·20–0·528 |
| $C$ | 0·0017–0·0127† | $3{\cdot}6 \times 10^{-7}$–$9{\cdot}2 \times 10^{-4}$ | 0·00065–0·0072 | 0·009–0·070 |
| $F$ | 1·83–2·11† | 0·70–0·99 | 1·28–2·48 | 1·10–2·16 |
| $Re$ | 6300–13 679 | N/A | 33 547–109 288 | 2548–12 546 |

* Gao and Abrahams' own data are listed in this column. These data were combined with the data of Song *et al.* in the analysis reported in their paper.
† Items are calculated from the data provided by the authors.

equivalent clear-water flows (Li and Abrahams, 1997). Sediment loads ranged up to 87·0 per cent of transport capacity and accounted for as much as 20·8 per cent of the flow resistance. Abrahams and Li (1998) established that $f_{bt}$ is a significant component of flow resistance in overland flow. Yet their study was limited in scope because it considered (1) a single slope and sediment size, (2) flows below transport capacity, and (3) fixed beds (Table I). Consequently, Abrahams and Li (1998) provided little information on the general magnitude and controls of $f_{bt}$ in flows transporting bed-load at capacity.

Song *et al.* (1998) also studied the bed-load transport resistance by conducting two series of experiments. The first involved hydraulically smooth flows transporting bed-load below capacity through a pipe, whereas the second involved hydraulically rough flows transporting bed-load at capacity through a flume with a plane mobile sediment-covered bed (Table I). Analysis of the combined data shows that the increase in flow resistance due to bed-load transport can be estimated by the equation

$$f/f_c = (1 + 30{\cdot}4CD_*^{\,0{\cdot}5})^{0{\cdot}92} \tag{2}$$

where $f$ is the friction factor of a sediment-laden flow, $f_c$ is the friction factor of an equivalent clear-water flow, $C$ is volumetric bed-load concentration, and $D_*$ is the dimensionless sediment diameter, given by

$$D_* = \left[\frac{\Delta g}{\nu^2}\right]^{1/3} D \tag{3}$$

where $\Delta = [(\rho_s - \rho)/\rho]$, $\rho_s$ is the density of the sediment (kg m$^{-3}$), $\rho$ is the density of the water (kg m$^{-3}$), $\nu$ is the kinematic viscosity of the water (m$^2$ s$^{-1}$), and $D$ is the median sediment diameter (m).

Gao and Abrahams (2004) combined their flume data with Song *et al.*'s flume data to develop a new equation for bed-load transport resistance in open-channel flows. Using dimensional analysis and multiple regression analysis, they obtained the equation

$$f_{bt} = 0{\cdot}048C^{0{\cdot}25}D_*^{\,0{\cdot}5}\left(\frac{h}{D}\right)^{-0{\cdot}75} \tag{4}$$

for relatively deep flows transporting gravel-sized sediment at low concentrations (Table I). The present study is an extension of Gao and Abrahams' (2004) study and is concerned with the controls of $f_{bt}$ for shallow overland flows transporting sand-sized sediments at relatively high concentrations. In contrast, the studies of $f_{bt}$ by Gao and Abrahams (2004) and Song *et al.* (1998) investigated deep open-channel flows transporting gravels at relatively low concentrations (see Table I). Gao and Abrahams identified three factors that control $f_{bt}$ in open-channel flows ($C$, $D_*$ and $h/D$). In overland flow, we identify another two factors that control $f_{bt}$. The ultimate goal of the study is to develop a parsimonious functional relation that may be used to predict $f_{bt}$.

## METHODS

As the flume used in this study has been described in previous publications (Abrahams and Li, 1998; Abrahams *et al*., 1998, 2001), only its main features are outlined here. The flume was 5·2 m long and 0·4 m wide with a smooth aluminium floor and Plexiglas walls. It consisted of two parts: a lower part 3·6 m long and a steeper upper part 1·6 m long. The floor of the lower part of the flume was covered with a layer of sand. Two well-sorted testing sands (ASTM C-109 and C-190) were used. For these sands $D$ equals 0·74 and 1·16 mm, while $D_{90}$ (particle size at which 90 per cent of sediment is finer) equals 0·81 and 1·46 mm, respectively. The lower part of the flume was inclined at a slope angle of $\beta = 2{\cdot}7$ or 5·5°. To correct for the effect of downslope component of gravity on sediment transport, sin $\beta$ was replaced by $S = \sin\beta \tan\alpha/[\cos\beta(\tan\alpha - \tan\beta)]$, in which $\alpha$ is the angle of repose, generally taken to be about 32° (Abrahams *et al*., 2001; van Rijn, 1993). Water entered the flume by overflowing from a head tank. The inflow rate $Q_w$ (m$^3$ s$^{-1}$) was measured by a flow meter located on the inlet pipe to the head tank.

Sand was supplied to the upper end of the flume by a continuously adjustable sediment feed system. During each experiment the sediment feed rate was adjusted to $Q_w$ so that the sand-covered bed experienced no perceptible scour or deposition. The purpose of the sediment feed system was simply to replace the sand being removed from the bed. It was assumed that the water would pick up all the sediment that it was capable of transporting before it reached the end of the flume. Govers and Rauws' (1986) finding that sediment transport capacities can be achieved in a 3-m-long flume lends credence to this assumption.

The volumetric sediment discharge $Q_s$ (m$^3$ s$^{-1}$) was determined by sampling the water–sediment mixture leaving the flume, weighing the water and sediment in each sample and converting the weights to volumes by multiplying by the water and sediment densities of $\rho_w = 1000$ kg m$^{-3}$, and $\rho_s = 2650$ kg m$^{-3}$, respectively. The volumetric sediment concentration $C$ was then obtained from

$$C = Q_s/(Q_w + Q_s) \tag{5}$$

Mean flow velocity $u$ was determined by a salt tracing technique described by Li and Abrahams (1997, 1999). Knowing $Q = Q_w + Q_s$ and $u$, mean flow depth $h$ was calculated from $h = Q/uw$, where $w$ is the flow width (i.e. 0·4 m). The kinematic viscosity of the water $\nu$ (m$^2$ s$^{-1}$) was obtained from its temperature. The study is based on 38 experiments (Table II) in which the overland flow is always supercritical and turbulent; that is, Froude number $F > 1$ and Reynolds number $Re \geq 2440$ (Savat, 1980), where

$$F = u/(gh)^{0{\cdot}5} \tag{6}$$

and

$$Re = 4uh/\nu \tag{7}$$

As this study is concerned with bed-load transport resistance, it is necessary to establish that sediment movement occurs mainly by bed-load transport. The proportion of the sediment load transported as bed-load $P_b$ was obtained by two methods. First, the suspended load expressed as a proportion of total load was calculated from Hu and Hui's (1996) equation 2 and subtracted from 1 to give $P_b$. Second, the bed-load transport rate was calculated using Abrahams and Gao's equation 15 (Abrahams *et al*. 2001) and expressed as a proportion of the measured total load. Hu and Hui's (1996) method indicates that $P_b$ ranges from 70·4 to 95·8 per cent, and averages 84·2 per cent, whereas Abrahams and Gao's method signifies that $P_b$ ranges from 70 to 100 per cent, and averages 82·7 per cent. Thus, it seems fair to conclude that in the present experiments sediment is transported predominantly as bed-load.

 

Table II. Experimental data for mobile beds

| $S$ | $D$ ($10^{-3}$ m) | $D_{90}$ ($10^{-3}$ m) | $u$ ($10^{-2}$ m/s) | $h$ ($10^{-2}$ m) | $C$ | Temp. (°C) | $\nu$ ($10^{-4}$ m$^2$/s) | $f$ | $f_g$ | $f_{bt}$ | $f_g/f$ (%) | $f_{bt}/f$ (%) | $Re$ | $\theta$ | $D_*$ | $h/D$ | $F$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0·05 | 0·7 | 0·8 | 30·6 | 0·48 | 0·015 | 19·4 | 0·0107 | 0·204 | 0·167 | 0·036 | 82·2 | 17·8 | 5451·4 | 0·198 | 17·9 | 6·4 | 1·42 |
| 0·05 | 0·7 | 0·8 | 25·3 | 0·39 | 0·017 | 19·7 | 0·0106 | 0·247 | 0·202 | 0·045 | 81·9 | 18·1 | 3759·8 | 0·164 | 18·0 | 5·3 | 1·29 |
| 0·05 | 0·7 | 0·8 | 24·4 | 0·37 | 0·015 | 19·8 | 0·0106 | 0·246 | 0·205 | 0·041 | 83·3 | 16·7 | 3333·6 | 0·151 | 18·0 | 4·9 | 1·30 |
| 0·05 | 0·7 | 0·8 | 28·0 | 0·45 | 0·019 | 19·8 | 0·0106 | 0·230 | 0·179 | 0·050 | 78·2 | 21·8 | 4452·8 | 0·175 | 18·0 | 6·1 | 1·38 |
| 0·05 | 0·7 | 0·8 | 34·9 | 0·53 | 0·020 | 19·8 | 0·0106 | 0·173 | 0·145 | 0·028 | 83·6 | 16·4 | 6941·1 | 0·219 | 18·0 | 7·1 | 1·54 |
| 0·05 | 0·7 | 0·8 | 37·8 | 0·57 | 0·023 | 19·8 | 0·0106 | 0·160 | 0·134 | 0·026 | 83·8 | 16·2 | 8155·5 | 0·238 | 18·0 | 7·7 | 1·60 |
| 0·05 | 0·7 | 0·8 | 40·0 | 0·62 | 0·017 | 19·8 | 0·0106 | 0·154 | 0·129 | 0·026 | 83·4 | 16·6 | 9310·3 | 0·256 | 18·0 | 8·3 | 1·63 |
| 0·05 | 0·7 | 0·8 | 41·9 | 0·64 | 0·020 | 19·8 | 0·0106 | 0·146 | 0·122 | 0·024 | 83·7 | 16·3 | 10143·7 | 0·267 | 18·0 | 8·7 | 1·67 |
| 0·05 | 0·7 | 0·8 | 42·2 | 0·70 | 0·022 | 19·9 | 0·0106 | 0·157 | 0·127 | 0·030 | 80·7 | 19·3 | 11156·9 | 0·291 | 18·0 | 9·4 | 1·61 |
| 0·05 | 0·7 | 0·8 | 43·7 | 0·70 | 0·023 | 19·9 | 0·0106 | 0·146 | 0·120 | 0·025 | 82·5 | 17·5 | 11497·0 | 0·289 | 18·0 | 9·4 | 1·67 |
| 0·05 | 0·7 | 0·8 | 44·1 | 0·73 | 0·020 | 19·8 | 0·0106 | 0·149 | 0·121 | 0·028 | 81·2 | 18·8 | 12072·5 | 0·302 | 18·0 | 9·8 | 1·65 |
| 0·05 | 0·7 | 0·8 | 47·3 | 0·67 | 0·020 | 19·7 | 0·0106 | 0·120 | 0·105 | 0·015 | 87·3 | 12·7 | 11986·1 | 0·280 | 18·0 | 9·1 | 1·84 |
| 0·05 | 0·7 | 0·8 | 20·0 | 0·34 | 0·011 | 19·7 | 0·0106 | 0·338 | 0·268 | 0·071 | 79·1 | 20·9 | 2548·1 | 0·141 | 18·0 | 4·6 | 1·10 |
| 0·05 | 1·2 | 1·5 | 28·1 | 0·50 | 0·012 | 5·7 | 0·0142 | 0·252 | 0·225 | 0·027 | 89·3 | 10·7 | 3935·2 | 0·132 | 23·2 | 4·3 | 1·27 |
| 0·05 | 1·2 | 1·5 | 31·9 | 0·58 | 0·011 | 5·6 | 0·0142 | 0·227 | 0·199 | 0·028 | 87·8 | 12·2 | 5163·4 | 0·153 | 23·2 | 5·0 | 1·34 |
| 0·05 | 1·2 | 1·5 | 33·8 | 0·68 | 0·018 | 5·6 | 0·0142 | 0·238 | 0·198 | 0·040 | 83·1 | 16·9 | 6164·8 | 0·172 | 23·2 | 5·9 | 1·31 |
| 0·05 | 1·2 | 1·5 | 40·6 | 0·62 | 0·009 | 5·6 | 0·0142 | 0·151 | 0·144 | 0·007 | 95·3 | 4·7 | 7072·2 | 0·165 | 23·2 | 5·3 | 1·65 |
| 0·05 | 1·2 | 1·5 | 40·2 | 0·70 | 0·012 | 5·5 | 0·0143 | 0·173 | 0·154 | 0·018 | 89·4 | 10·6 | 7861·9 | 0·185 | 23·1 | 6·0 | 1·54 |
| 0·05 | 1·2 | 1·5 | 40·6 | 0·77 | 0·018 | 5·4 | 0·0143 | 0·188 | 0·160 | 0·027 | 85·4 | 14·6 | 8778·0 | 0·205 | 23·1 | 6·7 | 1·47 |
| 0·05 | 1·2 | 1·5 | 45·4 | 0·81 | 0·021 | 5·4 | 0·0143 | 0·158 | 0·139 | 0·019 | 88·0 | 12·0 | 9777·3 | 0·204 | 23·1 | 7·0 | 1·61 |
| 0·11 | 1·2 | 1·5 | 25·6 | 0·42 | 0·043 | 7·2 | 0·0137 | 0·575 | 0·417 | 0·159 | 72·4 | 27·6 | 3147·2 | 0·249 | 23·7 | 3·6 | 1·26 |
| 0·11 | 1·2 | 1·5 | 30·3 | 0·47 | 0·037 | 6·7 | 0·0139 | 0·455 | 0·335 | 0·120 | 73·6 | 26·4 | 4078·1 | 0·275 | 23·6 | 4·0 | 1·41 |
| 0·11 | 1·2 | 1·5 | 34·3 | 0·55 | 0·047 | 6·3 | 0·0140 | 0·414 | 0·297 | 0·117 | 71·7 | 28·3 | 5338·0 | 0·321 | 23·4 | 4·7 | 1·48 |
| 0·11 | 1·2 | 1·5 | 44·2 | 0·58 | 0·040 | 5·5 | 0·0143 | 0·266 | 0·207 | 0·058 | 78·0 | 22·0 | 7206·5 | 0·342 | 23·1 | 5·0 | 1·85 |
| 0·11 | 1·2 | 1·5 | 52·8 | 0·61 | 0·051 | 5·6 | 0·0142 | 0·195 | 0·161 | 0·033 | 82·9 | 17·1 | 9014·4 | 0·358 | 23·2 | 5·2 | 2·16 |
| 0·11 | 1·2 | 1·5 | 31·1 | 0·46 | 0·047 | 5·5 | 0·0143 | 0·424 | 0·319 | 0·105 | 75·2 | 24·8 | 4003·4 | 0·270 | 23·1 | 4·0 | 1·47 |
| 0·11 | 1·2 | 1·5 | 41·5 | 0·50 | 0·056 | 5·5 | 0·0143 | 0·262 | 0·213 | 0·048 | 81·5 | 18·5 | 5307·5 | 0·269 | 23·1 | 4·4 | 1·87 |
| 0·11 | 1·2 | 1·5 | 47·1 | 0·55 | 0·058 | 5·4 | 0·0143 | 0·220 | 0·183 | 0·037 | 83·2 | 16·8 | 7186·3 | 0·321 | 23·1 | 4·7 | 2·04 |
| 0·11 | 0·7 | 0·8 | 32·3 | 0·42 | 0·050 | 19·2 | 0·0107 | 0·361 | 0·248 | 0·114 | 68·5 | 31·5 | 4825·3 | 0·390 | 17·8 | 5·7 | 1·59 |
| 0·11 | 0·7 | 0·8 | 22·6 | 0·32 | 0·051 | 19·8 | 0·0106 | 0·559 | 0·376 | 0·183 | 67·2 | 32·8 | 2566·4 | 0·295 | 18·0 | 4·3 | 1·28 |
| 0·11 | 0·7 | 0·8 | 34·6 | 0·65 | 0·056 | 19·9 | 0·0106 | 0·486 | 0·276 | 0·210 | 56·8 | 43·2 | 8030·8 | 0·602 | 18·0 | 8·8 | 1·37 |
| 0·11 | 0·7 | 0·8 | 38·6 | 0·49 | 0·066 | 20·0 | 0·0105 | 0·295 | 0·204 | 0·092 | 68·9 | 31·1 | 6788·1 | 0·455 | 18·1 | 6·7 | 1·76 |
| 0·11 | 0·7 | 0·8 | 39·9 | 0·55 | 0·067 | 20·1 | 0·0105 | 0·310 | 0·206 | 0·105 | 66·2 | 33·8 | 7880·0 | 0·511 | 18·1 | 7·5 | 1·71 |
| 0·11 | 0·7 | 0·8 | 37·9 | 0·63 | 0·069 | 20·1 | 0·0105 | 0·393 | 0·237 | 0·156 | 60·2 | 39·8 | 8555·4 | 0·584 | 18·1 | 8·5 | 1·52 |
| 0·11 | 0·7 | 0·8 | 39·2 | 0·65 | 0·067 | 20·1 | 0·0105 | 0·381 | 0·230 | 0·151 | 60·3 | 39·7 | 9163·3 | 0·605 | 18·1 | 8·9 | 1·55 |
| 0·11 | 0·7 | 0·8 | 47·0 | 0·75 | 0·059 | 16·8 | 0·0113 | 0·302 | 0·187 | 0·114 | 62·1 | 37·9 | 12546·0 | 0·689 | 17·2 | 10·1 | 1·74 |
| 0·11 | 0·7 | 0·8 | 42·7 | 0·78 | 0·063 | 16·9 | 0·0113 | 0·380 | 0·220 | 0·159 | 58·0 | 42·0 | 11846·5 | 0·716 | 17·3 | 10·5 | 1·55 |
| 0·11 | 0·7 | 0·8 | 43·4 | 0·73 | 0·070 | 17·1 | 0·0113 | 0·345 | 0·208 | 0·137 | 60·2 | 39·8 | 11282·4 | 0·671 | 17·3 | 9·8 | 1·63 |

## THE SAVAT ALGORITHM AND THE CALCULATION OF $f_{bt}$

In overland flows transporting bed-load on a plane bed, total flow resistance $f$ consists of grain resistance $f_g$ and bed-load transport resistance $f_{bt}$. Thus, $f_{bt}$ may be estimated from

$$f_{bt} = f - f_g \tag{8}$$

where $f$ is given by Equation 1 and $f_g$ is obtained from the Savat (1980) algorithm. In the following description of this algorithm, the subscript $g$ denotes the value of the associated variable when $f = f_g$.

The Savat (1980) algorithm was developed to compute the hydraulic properties of clear-water overland flows on plane beds. The algorithm applies to laminar as well as turbulent flows and was originally written in FORTRAN. A refined version of the algorithm was later written in PASCAL by G. Govers and is used in this study. The algorithm is based on an analysis by Savat (1980) of about 720 overland flows on smooth and grain-covered beds ranging in slope from 0·46° to 30·1°. Inputs to the algorithm are unit discharge $q$, water temperature, bed slope, and grain roughness $D_{90}$. Outputs include mean flow velocity $u_g$, mean flow depth $h_g$, and mean bed shear stress $\tau_g = \rho g h_g S$. Grain resistance $f_g$ is then calculated using $f_g = 8\tau_g/(\rho u_g^2)$. Extensive testing of the algorithm by comparing measured and predicted flow depths up to 0·02 m (e.g. Govers and Rauws, 1986; Rauws, 1988; Everaert, 1991; Takken and Govers, 2000) has confirmed its accuracy.

Because each flow is turbulent and fully rough according to Savat's (1980) criterion $D_{90} \geq 0{\cdot}394 S^{-0{\cdot}40} \nu$, the algorithm calculates $f_g$ by the following iterative procedure. An initial value of $U_g(n)$ is obtained by explicitly solving the Manning–Strickler equation

$$U_g(n) = \frac{0{\cdot}277 q^{0{\cdot}4} s^{0{\cdot}3}}{D_{90}{}^{0{\cdot}1}} \tag{9}$$

where $(n)$ denotes the $n$th iteration. Initial values of $h_g(n)$, $f_g(n)$ and $q(n)$ were then calculated from

$$h_g(n) = q(n)/U_g(n) \tag{10}$$

$$f_g(n) = \left[ 2\log \frac{h_g(n)}{D_{90}} + \frac{2{\cdot}1}{1 + 2{\cdot}1 \dfrac{D_{90}}{h_g(n)}} \right]^{-2} \tag{11}$$

$$q(n) = \left[ \frac{8gSh_g{}^3(n)}{f_g(n)} \right]^{1/2} \tag{12}$$

So long as $|\,[q(n)/q] - 1\,| > 0{\cdot}01$, a small increment was either added to or subtracted from $h_g(n)$, and $f_g(n)$ and $q(n)$ were recalculated using Equations 10 and 11.

Equation 11, which is the relation found by Savat between $f_g$ and $h_g/D_{90}$ for rough turbulent overland flow free of sediment, is graphed in Figure 1. Points representing the 38 sediment-laden overland flows investigated here are also plotted on the diagram. The plotted points all lie above the envelope curve defined by the equation, signifying that the value of $f$ for each flow is greater than the value of $f_g$ for the equivalent clear-water flow. The interval measured parallel to the vertical axis between the curve and each point equals $f_{bt}$.

The computed values of $f$, $f_g$ and $f_{bt}$ are reported in Table II along with the percentage of the total flow resistance due to bed-load transport resistance, $\%f_{bt} = 100 f_{bt}/f$. Table II shows that $\%f_{bt}$ ranges from 4·7 to 43·2 per cent and has a mean of 22·7 per cent (Figure 2). In contrast, the highest value of $\%f_{bt}$ obtained by Abrahams and Li (1998) was 20·8 per cent. The difference can be partly explained by the fact that the flows examined here were transporting sediment at capacity, whereas those investigated by Abrahams and Li (1998) were not. But

 

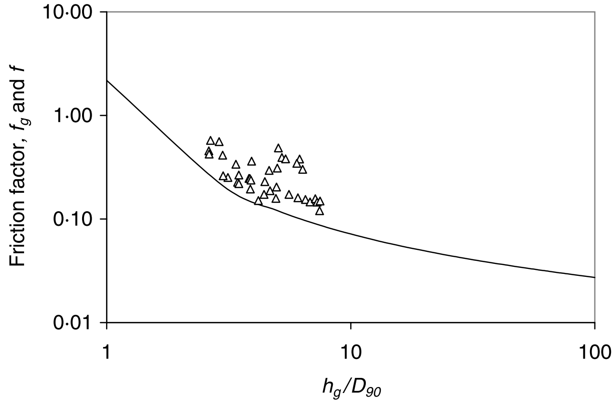


Figure 1. Relation between $f_g$ and $h_g/D_{90}$ for rough turbulent overland flow free of sediment

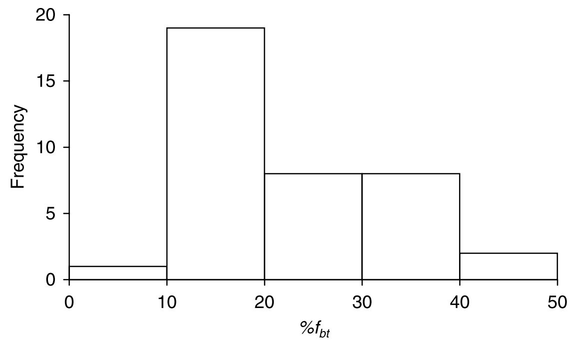


Figure 2. Distribution of $\%f_{bt}$

the main reason for the difference is that the present experiments include a wider range of hydraulic and sediment conditions than those performed by Abrahams and Li (1998) (see Table I). It is therefore concluded that in overland flow on plane beds, $f_{bt}$ is a much larger component of flow resistance than previously thought and that, consequently, greater attention needs to be paid to this source of resistance than has hitherto been the case.

## DIMENSIONAL ANALYSIS

In their dimensional analysis of bed-load transport resistance $f_{bt}$, Gao and Abrahams (2004) began with the basic variables $\rho$, $\rho_s - \rho$, $\mu$, $g$, $h$, $u_*$, $C$ and $D$. Notably absent from this list were $S$ and $u$. Given the fundamental nature of these two variables, we repeated the dimensional analysis with $S$ and $u$ included in the list. Thus, the initial functional relation is as follows

$$f_{bt} = \Phi(\rho,\ \rho_s - \rho,\ \mu,\ g,\ h,\ u_*,\ u,\ S,\ C,\ D) \tag{13}$$

where $\mu$ is dynamic viscosity of the fluid (N s m$^{-2}$), $u_* = (ghS)^{0.5}$ is shear velocity (m s$^{-1}$). Selecting $\rho$, $u_*$ and $D$ as the repeating variables and applying the $\Pi$-theorem yields

$$\pi_1 = \frac{\rho_s - \rho}{\rho} = \Delta \tag{14a}$$

$$\pi_2 = \frac{\mu}{\rho u_* D} = \frac{\nu}{u_* D} = \frac{1}{R_*} \tag{14b}$$

$$\pi_3 = \frac{gD}{u_*^2} = \frac{\rho g D}{\rho u_*^2} = \frac{\rho g D}{\tau} \tag{14c}$$

$$\pi_4 = \frac{h}{D} \tag{14d}$$

$$\pi_5 = \frac{u}{u_*} = \sqrt{\frac{u^2}{u_*^2}} = \sqrt{\frac{u^2}{ghS}} = \sqrt{\frac{u^2}{gh}}\sqrt{\frac{1}{S}} = FS^{-0\cdot5} \tag{14e}$$

where $R_*$ is grain size Reynolds number. Combining $\pi_1$, $\pi_2$ and $\pi_3$ produces

$$\frac{\pi_1 \times \pi_3}{\pi_2^2} = \frac{\Delta g D R_*^2}{\tau} = \frac{\Delta g D R_*^2}{ghS} = \frac{R_*^2}{\theta} = \frac{\Delta g D R_*^2}{u_*^2} = \frac{gD\Delta}{u_*^2}\left(\frac{u_* D}{\nu}\right)^2 = \frac{g\Delta D^3}{\nu^2} = D_*^3 \tag{14f}$$

where $\theta = hS/(D\Delta)$ is the dimensionless bed shear stress. Equation 13 therefore becomes

$$f_{bt} = \Phi\left(C, D_*, \frac{h}{D}, F, S\right) \tag{15}$$

and it can be seen that the inclusion of $u$ and $S$ in the set of basic variables leads to a functional relation that contains $F$ and $S$ in addition to the three dimensionless variables identified by Song *et al*. (1998) and Gao and Abrahams (2004).

## RELATIONS BETWEEN $f$, $f_{bt}$ AND $F$

Equation 15 indicates that $f_{bt}$ is a function of $F$. However, given

$$f = \frac{8ghS}{u^2} = \frac{8S}{F^2} \tag{16}$$

and

$$f_{bt} = f - f_g \tag{8}$$

it can be seen that $F$ is indirectly related to $f_{bt}$ through $f$. The question is whether $F$ is also directly related to $f_{bt}$ (i.e. independently of $f$) (Figure 3). To investigate this question, a stepwise regression was performed with $f_{bt}$ as the dependent variable and $f$ and $F$ as the independent variables. Both $f$ and $F$ entered the regression equation together accounting for 87·3 per cent of the variance in $f_{bt}$. The derived equation is

$$f_{bt} = 0{\cdot}5 f^2 F \tag{17}$$

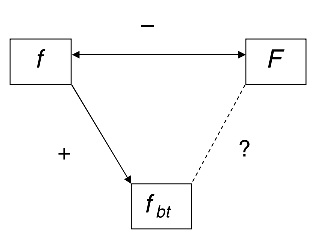


Figure 3. Relationship of $f_{bt}$ to $f$ and $F$

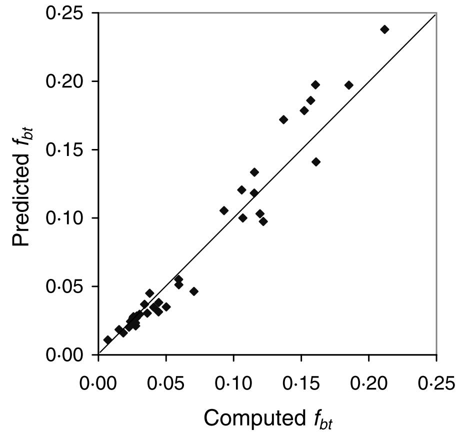


Figure 4. Comparison of computed $f_{bt}$ with $f_{bt}$ predicted by Equation 20

indicating that $F$ is related to $f_{bt}$ independently of $f$. The standardized regression coefficients for $f$ and $F$ are 0·992 and 0·174, respectively. These values signify that although $F$ affects $f_{bt}$ directly, the effect of $f$ on $f_{bt}$ is much larger. Note that Equation 17 may also be written as

$$f_{bt} = 0{\cdot}5f^2\ F = 0{\cdot}5\left(\frac{8S}{F^2}\right)^2 F = 32F^{-3}S^2 \tag{18}$$

indicating that $S$ as well as $F$ affects $f_{bt}$.

## FUNCTIONAL RELATION FOR $f_{bt}$

The functional relation given by Equation 15 may be written as a power product

$$f_{bt} = aC^b D_*^{\ c}\left(\frac{h}{D}\right)^d F^m S^n \tag{19}$$

and the coefficients $a$, $b$, $c$, $d$, $m$ and $n$ evaluated by performing a multiple regression analysis on the experimental data. The derived regression equation is

$$f_{bt} = 282{\cdot}5C^{0{\cdot}579}D_*^{\ -0{\cdot}800}\left(\frac{h}{D}\right)^{0{\cdot}25} F^{-3{\cdot}539}S^{1{\cdot}195} \tag{20}$$

with $R^2 = 0{\cdot}973$ and the standard error of estimate $SEE = 0{\cdot}06$ (Figure 4). The standardized (beta) coefficients for $C$, $D_*$, $h/D$, $F$ and $S$ are 0·447, −0·124, 0·093, −0·603, and 0·579, respectively. These coefficients indicate the relative importance of these five variables as controls of $f_{bt}$. Somewhat surprisingly, $F$ has the largest coefficient and thus emerges as the main control of $f_{bt}$.

## FACTORS CONTROLLING $f_{bt}$

In Equation 20, $f_{bt}$ is positively correlated with $C$ and negatively correlated with $D_*$. The former correlation is attributed to the frequency of grain collisions increasing as $C$ increases. The latter correlation can be ascribed to small particles being lifted higher into the flow than large particles. Thus, the small particles are subject to higher flow velocities, which transfer more momentum from the flow to the particles and ultimately to the bed.

 

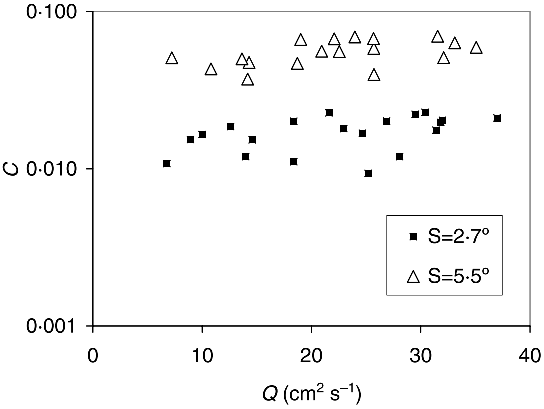


Figure 5. Relationship between $C$ and $Q$

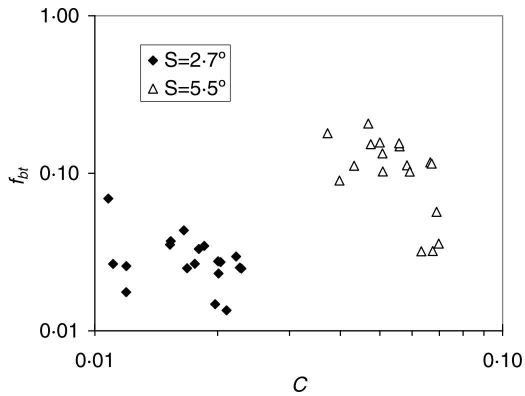


Figure 6. Relationship between $C$ and $f_{bt}$

Given that $D$ is proportional to $D_*$, which is held constant by the regression, the relation between $h/D$ and $f_{bt}$ reflects the behaviour of $h$, which is positively correlated with flow rate. Consequently, the positive relation between $h/D$ and $f_{bt}$ is ascribed to the frequency of collisions (i.e. momentum loss) increasing with flow rate.

Perhaps the most important finding of this study is the large negative exponent on $F$ signifying that in supercritical turbulent overland flow, $f_{bt}$ decreases rapidly as $F$ increases. This finding is explained in the following way. Momentum loss in bed-load transport occurs during two types of collision: grain-to-grain (GG) and grain-to-bed (GB). On average, more streamwise momentum is conserved in GG than in GB collisions, particularly at low shear stresses when many, if not most, grains colliding with the bed do not rebound. As $F$ increases, the relative frequency of GG collisions increases, and more momentum is conserved as the difference between incoming and outgoing grain velocities diminishes. In other words, as $F$ increases, a greater proportion of the bed becomes mobile, and a progressively larger proportion of collisions involve grains travelling at similar velocities, causing $f_{bt}$ to decline as $F$ increases.

Abrahams *et al*. (1998) reported that in overland flow $C$ increases with $S$, but is independent of the unit discharge. The same pattern is evident in the present data (Figure 5). Thus it can be seen that the positive correlation between $S$ and $f_{bt}$ reflects the positive relation between $C$ and $S$ (Figure 6).

## PARSIMONIOUS FUNCTIONAL RELATION FOR $f_{bt}$

Given that $S$ controls $f_{bt}$ through $C$, there is no need to include both $C$ and $S$ in a functional relation for $f_{bt}$. $C$ was therefore discarded from the set of independent variables in Equation 20, and a new equation was developed by regressing $f_{bt}$ on $D_*$, $h/D$, $F$ and $S$. The derived equation is

 

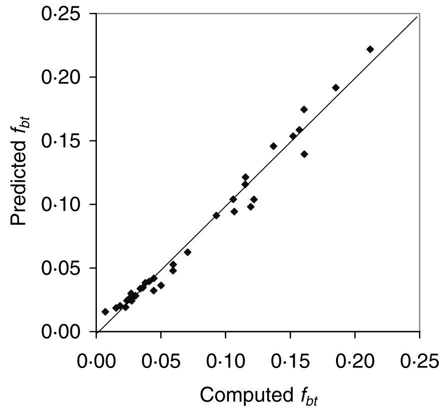


Figure 7. Comparison of computed $f_{bt}$ with $f_{bt}$ predicted by Equation 22

$$f_{bt} = 467{\cdot}74 D_*^{-1{\cdot}059}\left(\frac{h}{D}\right)^{0{\cdot}467} F^{-3{\cdot}348} S^{2{\cdot}0068} \quad C > 0 \tag{21}$$

with $R^2 = 0{\cdot}961$ and $SEE = 0{\cdot}072$. The standardized (beta) coefficients for $D_*$, $h/D$, $F$ and $S$ are $-0{\cdot}164$, $0{\cdot}175$, $-0{\cdot}571$ and $1{\cdot}00$, respectively, signifying that $S$ is the most important control of $f_{bt}$. The exponents on $D_*$, $h/D$, $F$ and $S$ are not significantly different from $-1$, $0{\cdot}5$, $-3$ and 2, respectively. Consequently, the exponents in Equation 21 were changed to these values, and the intercept was recomputed using non-linear regression. The functional relation therefore becomes

$$f_{bt} = 267{\cdot}3 D_*^{-1}\left(\frac{h}{D}\right)^{0{\cdot}5} F^{-3} S^{2} \quad C > 0 \tag{22}$$

with $R^2 = 0{\cdot}96$ and $SEE = 0{\cdot}072$ (Figure 7).

Although the $R^2$ value for this equation is smaller and the *SEE* value is larger than those for Equation 20, the differences are small (see Figures 4 and 7). This suggests that although Equation 20 contains five independent variables (controlling factors), only four are needed in the functional relation for $f_{bt}$. Inasmuch as Equation 22 does not contain $C$, it can be used to predict $f_{bt}$ without measuring the bed-load transport rate.

## CONCLUSIONS

This study explores the variation of $f_{bt}$ in supercritical overland flows on mobile plane beds. The study covers a wide range of hydraulic and sediment conditions. In the 38 flume experiments analysed here, $f_{bt}$ is calculated by subtracting grain resistance (obtained using Savat's algorithm) from total resistance $f$. The analysis reveals that $f_{bt}$ averages 22·06 per cent of total resistance.

This study is an extension of a recent study of $f_{bt}$ by Gao and Abrahams (2004). These authors found that $f_{bt}$ is controlled by three factors: $C$, $D_*$ and $h/D$. A new dimensional analysis in this study identifies two additional controlling factors, $F$ and $S$. Multiple regression analysis reveals that these five factors together explain 97 per cent of the variance of $f_{bt}$.

Further analysis indicates that in overland flow $S$ controls $f_{bt}$ through $C$. Consequently, the five controls of $f_{bt}$ can be reduced to four. In other words, when $S$ is viewed as a controlling factor, $C$ is redundant. Although the goal of the analysis is to identify and evaluate the relative magnitude of the factors controlling $f_{bt}$, the equation which characterizes the relation between these variables and $f_{bt}$ may also be used to predict $f_{bt}$. An advantage of this equation is that it permits $f_{bt}$ to be predicted without measuring bed-load transport rate.

 

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