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Estimation of the Philip Infiltration Parameters from Rainfall Simulation Data[1]

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ABSTRACT

A new, simple technique for estimating the parameters of the two-term Philip infiltration equation was developed and tested using field-measured data obtained in the northern Chihuahuan desert of New Mexico. The technique simultaneously provided information on the relationship between the Philip equation parameter A, and the field-measured hydraulic conductivity. The equation was reformulated as $I - cK_f = 1/2St^{-1/2}$, where I is the infiltration rate, S the sorptivity, t the time, K_f the field-measured final infiltration rate, and c, a coefficient relating K_f to the Philip parameter A. The final, steady infiltration rate measured in the field was used for the value of K_f. Regressions of $(I - cK_f)$ vs. $(1/2t^{-1/2})$ for values of c between 0 to 1 resulted in optimum c values for each treatment along with their corresponding S values. For the soils in this area, values for the coefficient c were sometimes outside the suggested range of 0.33 to 0.67, and were different for each study site. The regression analysis also showed that the value of S can be highly sensitive to changes in c. Using the values of S and c determined by the proposed method, a comparison was made between computed infiltration rates and measured infiltration rates. The results of this study showed that the prediction method provided adequate fits to field-measured data, and that the choice of an appropriate c factor is important in determining infiltration parameters from field data.

Additional Index Words: **infiltration prediction, sorptivity, saturated hydraulic conductivity, infiltration rate, runoff**

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IT IS OFTEN DESIRABLE to describe infiltration of water into soil with a small number of parameters that can then be used as inputs to hydrologic models. These parameters are often the coefficients of algebraic infiltration equations that relate the cumulative infiltration, or the infiltration rate, to time. Philip (1957a) showed that a complex infiltration equation can be expressed by a simpler, rapidly converging power series in $t^{1/2}$. The first two terms of this series may be retained as a concise infiltration equation of the form

$$i = St^{1/2} + At \quad [1]$$

where i is the cumulative infiltration and t is time since ponding on a uniform soil (Philip, 1957b). In this equation, S is the sorptivity and A is a constant reflecting an essentially steady rate at long times.

One difficulty with using the Philip equation is the uncertainty in the estimation of the parameter A. Philip (1969) noted that this equation is inappropriate for long-time experiments because in the limit, as t goes to infinity, $di/dt = K_o$, where K_o is the saturated hydraulic conductivity of the soil. However, A may not be equal to K_o, and there is no general analytical relationship between the two (Smiles and Knight, 1976; Collis-George, 1977). This problem has also been discussed by Youngs (1968), Philip (1969), Swartzendruber and Youngs (1974), and Fleming and Smiles (1975).

Several investigators have empirically correlated A with K_o. Talsma (1969), using ring infiltrometer data, found that $A = 0.36\ K_o$. On the other hand, comparison of the two-term Philip Eq. [1] with the Green and Ampt equation suggests that $A = 0.67\ K_o$ (Youngs, 1968). Whisler and Bouwer (1970) calculated sorptivity with the Philip equation using various relationships between A and the hydraulic conductivity of the soil. The best fit for cumulative infiltration was obtained when $A = 0.75K_r$, where K_r was the hydraulic conductivity of the wetted zone that may not be equal to K_o.

To determine the parameters S and A of the Philip equation, least squares techniques can be used, which often results in negative values of the constant A (Skaggs, et al., 1969; Jaynes and Gifford, 1981). Thus, the use of the Philip equation with parameters that have been determined by regression may cause predicted infiltration rates to be too low for times greater than the duration of the experimental test (Watson, 1959; Skaggs, et al., 1969). A different technique is needed for predicting these parameters.

In several studies, the value of A was calculated as $0.33\ K_o$, where K_o was determined from the steady infiltration rate measured in the field after 1 h or more (Sharma et al., 1980; Chong, 1983). Infiltration rates were then calculated using values of sorptivity measured in the field, or were calculated from simple algebraic equations. However, independent measurements of K_o indicate the steady state infiltration rate measured in the field is not equal to the saturated hydraulic conductivity (Whisler and Bower, 1970; Sharma et al., 1980), and A may not be equal to 0.33 K_o.

The objective of this paper is to develop a new, simple technique for estimating the parameters of the Philip infiltration equation while simultaneously determining the relationship of the parameter A to the field measured steady-state infiltration rate K_o.

Study Area and Methods

Infiltrometer studies were conducted on a small watershed located in the northern Chihuahuan Desert, 25-km northeast of Las Cruces, NM. The area is within the long-term ecological research site of New Mexico State Univ. Studies were carried out on three vegetation-soil complexes common to the area. They are referred to here as the upper, middle, and lower sites, in reference to their physiographic position in the watershed. The upper site, part of the Aladdin series (Torriorthentic Haplustolls, coarse-loamy, mixed, thermic) (Nash, unpublished data) contains predominant vegetation of black gramma grass (*Bouteloua eripoda*). The middle site, part of the Onite series (Typic Haplargids, fine-loamy, mixed, thermic), supports a cover of snakeweed shrub (*Gutierrezia sarothrae*) as its predominant vegetation. The lower site, part of the Bucklebar series (Typic Haplargids, fine-loamy, mixed, thermic), contains a cover of burro grass

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(*Scleropogon brevifolius*) as its predominant, and almost exclusive, vegetative cover.

The upper and middle sites were divided into vegetative plots and interspace plots; that is, plots located directly over the predominant vegetation, and plots located between the predominant vegetation, respectively. On the lower site, vegetation was sparse but continuous, and consequently only one set of plots was sampled.

Data were collected using a modified Purdue sprinkling infiltrometer (Bertrand and Parr, 1961), with modifications as discussed by Seiger (1984). Simulated rainfall was applied to 1-m^2 plots separated from the surrounding soil by steel frames inserted to a depth of approximately 6 cm. The rainfall intensity delivered by the simulator varies with orientation of the nozzle assembly with respect to the plot frame. Because of this variation, rainfall intensities were measured before and after each simulator run. This was done by placing an Al cover over the plot, and measuring the total runoff from the cover over time. Three rate measurements were made before and after each run, and the mean intensity was calculated. Antecedent soil moisture effects on infiltration were determined by conducting a second run approximately 24 h after the initial run, followed by a third run 45 min after the second one. The terms dry run, wet run, and very wet run, respectively, are used in this paper to describe the three antecedent moisture conditions. Each site, vegetation zone, and antecedent moisture level will be referred to as a treatment in the subsequent analysis. Infiltration measurements were made on several plots per treatment to provide for replication of the data (Table 1). Runoff from the plots was collected in a trough, which was emptied with a small centrifugal pump at constant time intervals. Water removed from the trough was measured volumetrically to provide values of incremental runoff rate. Runoff measurements were continued until an apparent steady runoff rate was maintained for three or more measurement intervals.

Infiltration rate is calculated as rainfall rate minus runoff rate, after corrections for interception loss and depression storage have been made. In this study, it was assumed that the interception and depression storage had been satisfied at the time of runoff initiation. Infiltration rate was then calculated as rainfall rate from time of runoff to time of measurement, minus the runoff rate in this period. Rainfall applied before the time of runoff initiation was not included in the calculation of infiltration rate. This approach provides a lower, but more realistic, value of infiltration rate than would be obtained if initial abstractions were not considered. This also insured that only post-ponding infiltration was considered in this analysis. To determine the steady infiltration rate, the arithmetic average of the last three steady-state infiltration rate values was calculated. These rate values correspond to the portion of the infiltration rate curve where infiltration has become constant.

To estimate the parameters of the Philip equation (Eq. [1]), the equation was recast as

$$I - cK_f = \tfrac{1}{2}St^{-1/2} \quad [2]$$

where I is the infiltration rate, K_f is the final field-measured hydraulic conductivity, and c is a coefficient relating K_f to the Philip parameter, A. The steady infiltration rate determined from the data was used as an estimate of K_f. This requires an assumption of a unit hydraulic gradient with depth in the profile during the final infiltration measurements. For this reason, readings are continued until runoff rates are constant for several measurement intervals. This method is discussed in detail by Chong and Green (1979). However, as indicated earlier, K_f is not always equal to the saturated hydraulic conductivity, again emphasizing the need for determination of the factor c. Using values of c ranging from 0 to 1, a regression of $I - cK_f$ vs. $1/2t^{-1/2}$ was performed for each rainfall simulator run. Sorptivity was calculated for each run as the slope of the best fit line. The optimum value of c was chosen such that the total mean square error for all regression lines and for each treatment was minimized. This provided an optimum ratio of K_f to A for each site, and the best estimate of sorptivity for each run.

Table 1. Optimum values of c and means and standard errors for sorptivity, S and final infiltration rate, K_f, for each site, vegetation zone, and antecedent moisture condition. Standard errors are in parenthesis.

Site†	Zone‡	AMC§	Initial moisture (0–10 cm)	No. reps	c	S, cm/h$^{1/2}$	K_f, cm/h
U	V	D	0.011 (0.004)	4	0.95	0.88 (0.15)	7.52 (0.39)
		W	0.093 (0.006)	4	0.82	1.24 (0.14)	5.78 (0.69)
		VW	0.139 (0.007)	3	0.94	0.37 (0.05)	5.09 (1.17)
U	I	D	0.022 (0.004)	3	0.93	1.09 (0.08)	9.39 (0.51)
		W	0.080 (0.007)	3	0.76	2.48 (0.09)	6.79 (0.32)
		VW	0.139 (0.03)	2	0.80	1.71 (0.14)	6.07 (0.59)
M	V	D	0.022 (0.010)	4	0.99	0.12 (0.03)	9.39 (0.41)
		W	0.096 (0.004)	4	0.97	0.50 (0.11)	9.11 (1.12)
		VW	0.145 (0.002)	3	0.88	1.52 (0.24)	6.88 (0.96)
M	I	D	0.014 (¶)	3	0.74	3.53 (¶)	8.48 (¶)
		W	0.095 (0.003)	3	0.84	2.40 (0.23)	12.67 (6.73)
		VW	0.140 (0.003)	2	0.58	3.37 (1.50)	8.10 (4.14)
L	I	D	0.099 (0.009)	4	0.00	1.93 (0.22)	1.78 (0.36)
		W	0.141 (0.006)	4	0.37	1.04 (0.13)	1.73 (0.21)
		VW	0.184 (0.017)	4	0.38	0.72 (0.04)	1.19 (0.16)

† U = upper; M = middle; L = lower.
‡ V = vegetative plots; I = interspace plots.
§ Antecedent moisture condition: D = dry run; W = wet run; VW = very wet run.
¶ Only one replication could be used to calculate value.

RESULTS AND DISCUSSION

Figure 1 shows a plot of $I - cK_f$ vs. $1/2t^{-1/2}$ for one run on the upper site, vegetative zone with wet antecedent moisture condition. The solid line represents the best fit, 0 intercept, straight line through the experimental data points, for a value of c of 0.82. Regression lines with 0 intercepts were used in the analysis because (i) there were no significant intercepts and (ii) the 0 intercept is physically easier to relate to the data. A value for c of 0.82 provided the lowest total overall mean square error for this treatment and thus was considered the optimum value. Equation [2] under-

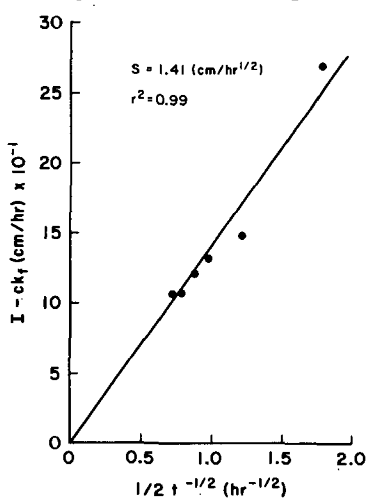

Fig. 1. $I - cK_f$ vs. $1/2t^{-1/2}$ for a 0 intercept fit; upper site; vegetative zone; wet run; c = 0.82.

predicted the infiltration time relationship at small times (large $1/2t^{-1/2}$), and slightly overpredicted it at large times for this simulator run. For practical purposes, however, the overall fit appears to be quite good as indicated by an r^2 of 0.99. The slope of this line provides a value of sorptivity that can then be used in the Philip equation to predict infiltration.

Table 1 gives the optimum values of c for all treatments, along with the average values of S and K_f. As can be seen in this table, the optimum value of c is different for each treatment and is sometimes outside the range of 0.33 to 0.67 suggested in the literature. Thus, it appears that the factor c should be determined separately for each area under study.

The difference in c factors among different areas can be partly explained by examining Eq. [2]. For the upper and middle sites, which have sandier soils, infiltration rates are largely controlled by K_f and, therefore, c values are relatively high. For the lower site, the infiltration rates are not as strongly controlled by K_f and, therefore, c values are relatively low. For the dry run on the lower site, a value of 0.00 was found for the c factor, indicating that the influence of K_f was very small (none) for this treatment.

It also can be seen in Table 1 that the values of S and K_f vary among treatments. Analysis of variance tests show that there are significant differences in S by site, vegetation zone, and antecedent moisture condition, while there are significant differences in K_f by site and antecedent moisture condition only. The standard errors of the estimates of S and K_f were much higher for the middle site interspace areas than for any other treatment. Although there were three replications for this area, infiltration rates were greater than rainfall rates during the entire simulator run for two of the three replications on the dry run and for one of the three replications on the wet run. Because no runoff occurred from these plots, K_f and S could not be calculated for these plots. The small sample size and large spatial variability for the middle site caused the high coefficient of variation for these treatments.

Because a value of 0.33 has been used for c in many studies, the sensitivity of sorptivity to changes in c was investigated. Graphs of average sorptivity vs. c factor for each of the three main sites are plotted in Fig. 2. The value of S is highly sensitive to the choice of c for the upper and middle site, but it is not as sensitive to c on the lower site. For example, an increase in c from 0.33 to 0.67 at the upper site reduces the average S from 4.98 to 2.73, a 45% decrease. At the middle site, an increase in c from 0.33 to 0.67 reduces S from 6.96 to 3.68, a 47% decrease. The same change in c at the lower site reduces S from 1.10 to

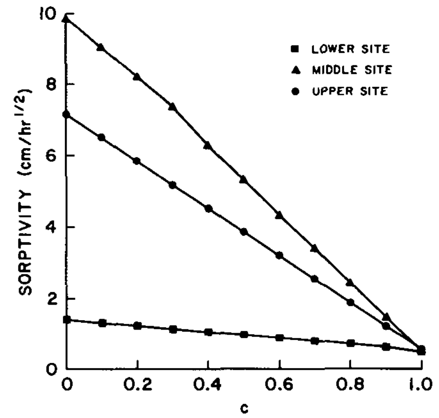

Fig. 2. Sensitivity of sorptivity to choice of c value.

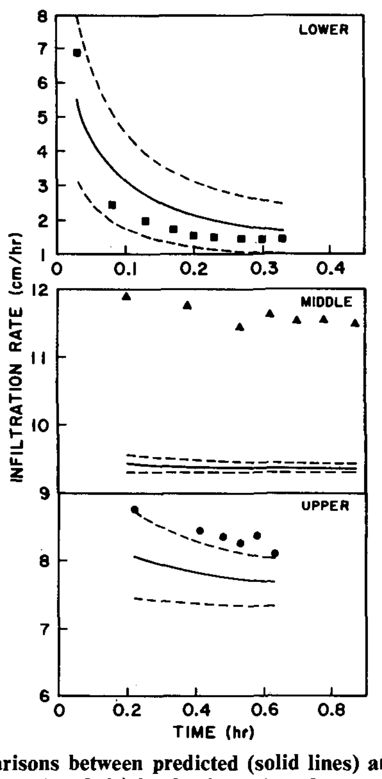

Fig. 3. Comparisons between predicted (solid lines) and measured infiltration rates (symbols) for the three sites; dry runs. The dashed lines are the limits for two standard deviations.

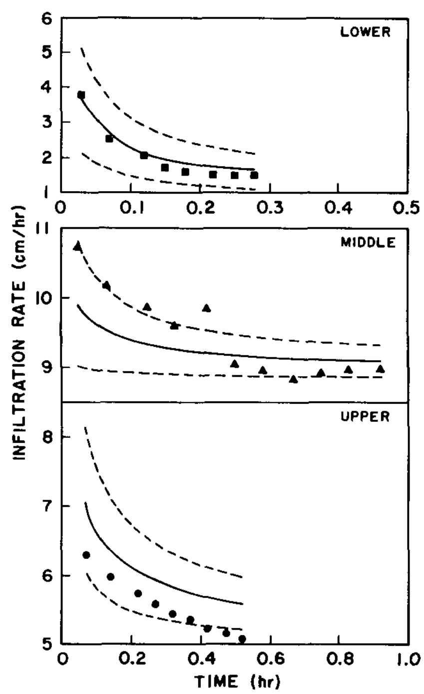

Fig. 4. Comparisons between predicted (solid lines) and measured infiltration rates (symbols) for the three sites; wet runs. The dashed lines are the limits for two standard deviations.

0.81, a 26% decrease. The difference in sensitivity to c between the upper two sites and the lower site can be explained by the same reasoning used to explain the difference in the value of c for different areas. For the upper and middle sites, infiltration rates are largely controlled by K_f and are, therefore, strongly affected by the choice of c. For the lower site, infiltration rates are not as strongly controlled by K_f, and a change in c does not substantially change the value of infiltration and, therefore, sorptivity.

The utility of this technique was tested by comparing predicted infiltration rates with infiltration rates measured on plots in the field that were not used to determine the values of c, S, and K_f. The average values of c, S, and K_f that were computed for each treatment were used to calculate the predicted infiltration rates. Examples of comparisons between predicted infiltration rates and measured infiltration rates for dry and wet antecedent moisture conditions are shown in Fig. 3 and 4, respectively. Also shown are the limits for infiltration rates when S is varied plus and minus two standard deviations from its mean while K_f is held constant at its average value. For the dry runs, the data provides good predictions for the upper and lower sites but a somewhat poor prediction for the middle site. For the wet runs, the data provides good predictions for all three sites with the middle site again appearing to have the poorest prediction. In general, infiltration rates on dry soils are more variable than on wet soils, and prediction is, therefore, somewhat more complicated. Considering this fact, the prediction technique developed here provides encouraging results. The poor prediction for the middle site is probably due to the high spatial variability found for this site, as was previously discussed. Predicted infiltration rates for the very wet run, although not shown here, provided similar results as for the wet run.

To further investigate the utility of the prediction technique, the predicted vs. measured infiltration rates were plotted in relation to a 1:1 line. The data from the dry runs are plotted in Fig. 5 and the data from the wet runs in Fig. 6. It can be seen in these figures that a fair agreement was obtained between predicted infiltration rates and measured infiltration rates. Again, predictions for the wet runs are somewhat better than for the dry runs. The relative error for each predicted value was determined by dividing the absolute difference between the measured and predicted value by the measured value. The relative error for the dry runs ranged from 6.4 to 44.3% with an average of 20.6%. The relative error for the wet runs ranged from 1.5 to 15.6% with an average of 6.4%. The overall average for both runs was 12.6%. The average relative error of 20.6% for the dry runs appears to be high, but again, the high spatial variability of the middle site contributed significantly to this error. The average relative error of 6.4% for the wet runs is quite good, considering the many sources of error inherent in any field experiment. For this particular experiment, one source of error may be the assumption that interception loss and depression storage were satisfied at the time of runoff initiation, i.e., only infiltration loss was occurring. This may be especially significant on the upper and middle vegetative sites, where vegetative cover was approximately 50%. Another source of error is the use of the value of the steady infiltration rate as a measure of K_f, because the assumption of a unit hydraulic gradient in the field may not have been satisfied. However, considering these sources of error, the parameters obtained by the above technique appear to be sufficiently accurate for prediction of infiltration rates on similar field sites.

SUMMARY AND CONCLUSIONS

This study produced a new, simple technique for evaluating the parameters of the Philip infiltration equation. The equation was written as $I = 1/2St^{1/2} + cK_f$, where I is the infiltration rate, S is the sorptivity, K_f is the final, field-measured hydraulic conductivity, and c is a coefficient relating K_f to the Philip parameter A. The final, steady infiltration rate measured in the field was used for the value of K_f. Sorptivity was calculated by graphing $(I - cK_f)$ vs. $(1/2t^{-1/2})$ for values of c ranging from 0 to 1. The value of c was chosen such that the total mean square error for the regression lines was minimized for each treatment, i.e., for each particular site, vegetative zone, and antecedent moisture condition.

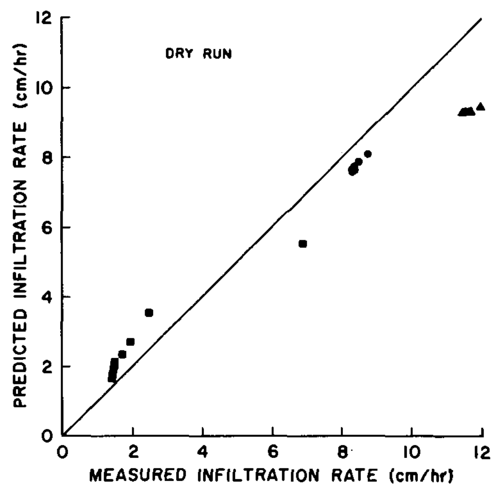

Fig. 5. Predicted vs. measured infiltration rates relative to a 1:1 line for the dry runs of Fig. 3. Squares denote the lower site, triangles the middle site and dots the upper site.

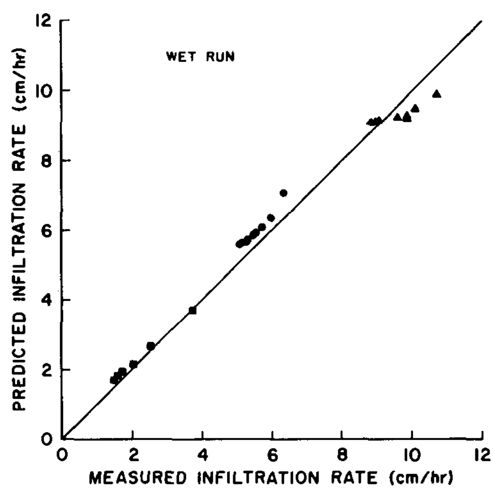

Fig. 6. Predicted vs. measured infiltration rates relative to a 1:1 line for the wet runs of Fig. 3. Squares denote the lower site; triangles the middle site and dots the upper site.

The results of this study show the optimum value of *c* is sometimes outside the range of 0.33 to 0.67 suggested in the literature. In addition, the optimum value of *c* is site specific and should be determined for each soil-vegetation complex under study. The value of sorptivity calculated for each site is dependent upon choice of *c*. A sensitivity analysis of *S* showed sorptivity was highly sensitive to choice of *c* and sensitivity varied among the three sites.

The calculated parameters were assessed by evaluating their ability to predict infiltration rates on plots in the field adjacent to the experimental plots. Adequate fits to measured infiltration data were obtained using the average calculated parameters for each treatment; however, the dry runs were not as accurately predicted as the wet runs. The average relative error between predicted and measured infiltration rates was 20.6% for the dry runs and only 6.4% for the wet runs. Large spatial variability on the middle site contributed significantly to the high relative error for the dry runs. This limitation must be considered when using any prediction technique in field situations. Overall, however, considering the accuracy of field measured data, the method discussed appears to be a useful approach for obtaining values for the Philip infiltration parameters that can then be used for watershed modeling purposes.

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